

Fig. 6. A model of the first Brillouin zone of a body-centered cubic lattice. The sphere occupies half the volume of the zone.

situation is illustrated in Fig. 6, which shows a model of the first Brillouin zone of a body-centered cubic lattice containing a sphere whose volume is just half that of the zone. Figure 7 shows a two-dimensional square lattice, its two-dimensional Brillouin zone and the Fermi "circle" having an area of just half that of the zone. In Fig. 7 (a, b, c, and d) I have shown, purely schematically, the progressive distortion of the Fermi surface; in d this surface is in marked contact with the Brillouin zone. In general, distortion of the Fermi surface causes those regions which are nearest the zone boundaries to become even closer.

A great deal is now known about the Fermi surfaces of the noble metals from

a variety of techniques which give direct information about the shape and other features of the Fermi surface (16). These methods agree in showing that the Fermi surfaces in copper, silver, and gold all touch the zone boundary. About the alkali metals we have as yet no direct evidence, but indirect evidence suggests that the Fermi surfaces of sodium and potassium are nearly spherical, that the Fermi surface of rubidium is somewhat distorted, and that the Fermi surfaces of lithium and cesium are much more distorted, perhaps touching the Brillouin zone boundary (17, 18). The effects of such distortion on electrical resistivity are discussed later.

When an electric field is applied to a metal the conduction electrons are accelerated and the whole Fermi surface begins to move in the direction of the field (see Fig. 8). The electrons, however, are prevented from continuous acceleration in the field by collisions with phonons (we are considering only the ideal resistivity), and the Fermi-surface movement is almost vanishingly small. The effect of the distortion of the Fermi surface on the scattering of electrons by phonons is a difficult theoretical problem, and detailed studies have only recently been made (19). One of the most important effects arises from a type of scattering process called an "Umklapp" process, which gives rise to large angle scattering of the electrons.

First consider a typical *normal* scattering process in which an electron of

wave vector k is scattered by a phonon of wave vector q into a new state of wave vector k' ; k , k' , and q are related by the vector condition that $k - k' = q$. Moreover, the phonon energy at a normal and low temperatures is very small compared with the Fermi energy of the electrons. Since only those electrons near the Fermi level have neighboring unoccupied states into which they may be scattered, the scattered electron must both start and end effectively at the Fermi surface. Figure 8 shows the geometry of a normal scattering process. As mentioned earlier, the Brillouin zone governs the behavior of all kinds of waves that can propagate through the metal, including lattice waves; the biggest wave vector that a phonon can have is one which reaches from the center of the zone to the zone boundary. This therefore limits the angle through which an electron may be scattered in a normal process even at the highest temperatures. At low temperatures, where only low energy phonons (having therefore small wave vectors) are excited, the angle of scattering is even further limited in such processes.

An Umklapp process may be interpreted as one in which the electron scattered by a phonon and also undergoes a Bragg reflection. In vector terms the well-known Bragg condition is represented by the equation $k' - k = R$ where R is a reciprocal lattice vector. In Fig. 7 the vectors R' and R'' are two reciprocal lattice vectors for the simple square lattice. Thus, in an Umklapp process the vector condition

$$k' - k = q$$

is replaced by

$$k' - k = q + R$$

where R is a reciprocal lattice vector. Such a process is illustrated in Fig. 9. Its importance lies in the fact that because the large vector R enters in the process, it makes possible scattering at wider angles than can occur in a normal process. This can also be seen by a geometrical construction. Equation 8 may be rewritten as

$$k + R = k' - q$$

and we begin by representing graphically all the possible vectors $k + R$. Since the k vectors of all electrons which can be scattered must lie on the Fermi surface, the vectors $k + R$, for example, must lie on the same surface displaced by the vector R ; the same

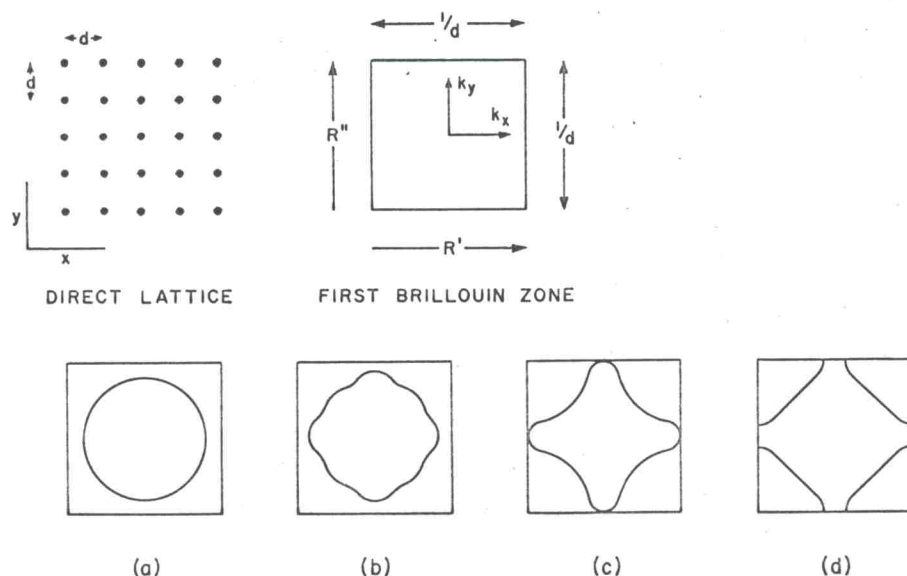


Fig. 7. A two-dimensional square lattice and the corresponding first Brillouin zone. (a) The Fermi "circle" corresponding to one electron per atom; (b), (c), and (d), progressive distortions of the Fermi surface (schematic).